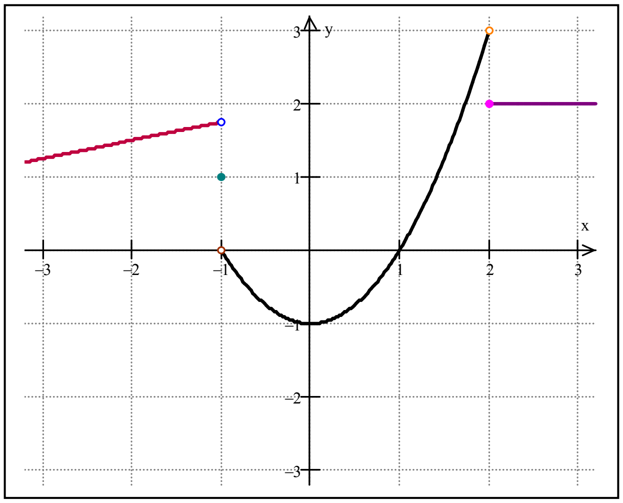
Name: PD: Date:

Quiz 2 – Continuity, Some basic limit ideas, Limit definition of derivative



1) $\lim_{x\to -1} f(x) = ?$ Justify your answer.

This is asking for the 2-sided limit as x-> -1. So – we have to ask about the two 1-sided limits (from left and right). From the left, the function comes into the square from the left, and approaches a value of about 1.75 as the *x*-coordinate of the points approaches -1. From the right we follow the parabola down, and then up, and **that** curve approaches a *y*-value of 0 as the *x*-coordinate approaches -1. Since the two values are different (1.75 and 0), the limit **does not exist**.

2) $\lim_{x\to +1} f(x) = ?$ Justify your answer.

Both one-sided limits approach a *y*-value of 0 as *x* approaches 1. So the limit = 0. (and fn is cont.)

---

3) The graph of $y = \dfrac{x^2-9}{3x-9}$ has:

(A) a vertical asymptote at x=3 (B) a horizontal asymptote at y=1/3
(C) a removable discontinuity at x=3 (D) an infinite discontinuity at x=3
(E) none of these

Asymptotes and discontinuities suggest limit things … so let's consider this from that perspective. The function would be defined and continuous **except** when the denominator is 0. So (solving) the only problem is at *x* = 3. We notice *3* choices with an *x* value of 3, so let's consider the limit.

$\lim_{x\to 3} \dfrac{x^2-9}{3x-9} = [\dfrac{0}{0}] = \lim_{x\to 3} \dfrac{(x+3)(x-3)}{3(x-3)} = \lim_{x\to 3} \dfrac{(x+3)}{3} = \dfrac{6}{3} = 2.$ Choices (A) and (D) both require the limit to be infinite, so we can rule them out. Choice (C) looks really good since we can remove the discontinuity cause by the hole. Choice (B) requires us to take the limit as x approaches +/- infinity. We do that and notice that the limit is +/- infinity and **not** a number (which would be required if I had a horizontal asymptote). So **(C)** is the correct answer.

4) $\lim_{x\to 0} \dfrac{x}{x} =$ (A) 1 (B) 0 (C) ∞ (D) -1 (E) does not exist

When we substitute we get 0 over 0 … so WE DON'T KNOW ANYTHING! Can we cancel anything? YES!

So we get $\lim_{x\to 0} \dfrac{x}{x} = \lim_{x\to 0} 1 = 1$. **(A)**

5) Let $f(x) = \begin{cases} \dfrac{x^2-1}{x-1}; & x \neq 1 \\ 4 \quad ; & x = 1 \end{cases}$. Which of the following statements are true?

I. $\lim_{x\to 1} f(x)$ exists

II. *f(1)* exists.

III. *f* is continuous at *x=1*

(A) I only (B) II only (C) I and II (D) all of them (E) none of them

There are two ways to live: you can live as if nothing is a miracle;
or you can live as if everything is a miracle. - Einstein

Name: PD: Date:

$\lim\limits_{x\to 1^-} f(x) = \lim\limits_{x\to 1^-} \frac{x^2-1}{x-1} = [\frac{0}{0}] = \lim\limits_{x\to 1^-} \frac{(x-1)(x+1)}{x-1} = \lim\limits_{x\to 1^-}(x+1) = 2$. And that's the same limit from the right hand side. So I is true.

$f(1) = 4$ … so II is true.

Since the numbers are not equal, III is false.
Therefore, **(C)** is the correct choice.

6) If $f(x) = \begin{cases} \frac{x^2-x}{2x}, & x \neq 0 \\ k, & x = 0 \end{cases}$, and if f is continuous at $x = 0$, then k=

(A) -1 (B) -1/2 (C) 0 (D) +1/2 (E) 1

Another continuity question. Let's find the limit …. $\lim\limits_{x\to 0} \frac{x^2-x}{2x} = \lim\limits_{x\to 0} \frac{x(x-1)}{2x} = \lim\limits_{x\to 0} \frac{(x-1)}{2} = \frac{-1}{2}$. Notice this is the two-sided limit. Some of you got confused about the way these piecewise functions were defined. Don't be! For example … what if I wanted to describe the function pieces that consisted of $\frac{x^2-x}{2x}$ for *x* values bigger than 0; the value '4' when *x* = 0, and again $\frac{x^2-x}{2x}$ for *x* values less than 0.
The function definition above is the most compact (most 'efficient') was of describing that function….
Anyway … If *f* is continuous then the value of the function (*k*) has to equal the limit, so *k* must be -1/2.
(B) is the correct answer …

7) Using this definition of the derivative:
$f'(x) = \lim\limits_{h\to 0} \frac{f(x+h)-f(x)}{h}$, determine the derivative of $f(x) = x^2 + 14$,

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{(x+h)^2 + 14 - (x^2+14)}{h} = \lim_{h\to 0} \frac{x^2+2hx+h^2+14-x^2-14}{h} = \lim_{h\to 0}(2x+h) = 2x$$

8) $\lim\limits_{x\to\infty} \frac{1}{x} = 0$. Write a sentence or two to a non-mathematician explaining what this means.

There were lots of good answers to this question, and reading them is enlightening (for me). Our understanding of limits begins with the idea of substituting numbers into the function that are closer and closer to the desired *x* value, and observing what happens to the corresponding *y* values. So … it examines the relationship between *x* and *y* in a certain way – namely how the *x* and *y* change together.

A more … evolved? … understanding would highlight that relationship more, and rely on the numeric calculations less.

A great answer would try and avoid mathematical terms like infinity and perhaps even limit; get in the idea that we're concerned with what happens as *x* gets larger and larger, include the idea that our observed *x* values never actually include the point in question, and maybe even the idea that the 'limit' number is a *y* number.

If your score was less than 19 out of 24, I entered it in eSchool as 18.yourscore. Learn this – most of these questions were pretty easy. Gifts won't go on forever …. !

There are two ways to live: you can live as if nothing is a miracle;
or you can live as if everything is a miracle. - Einstein